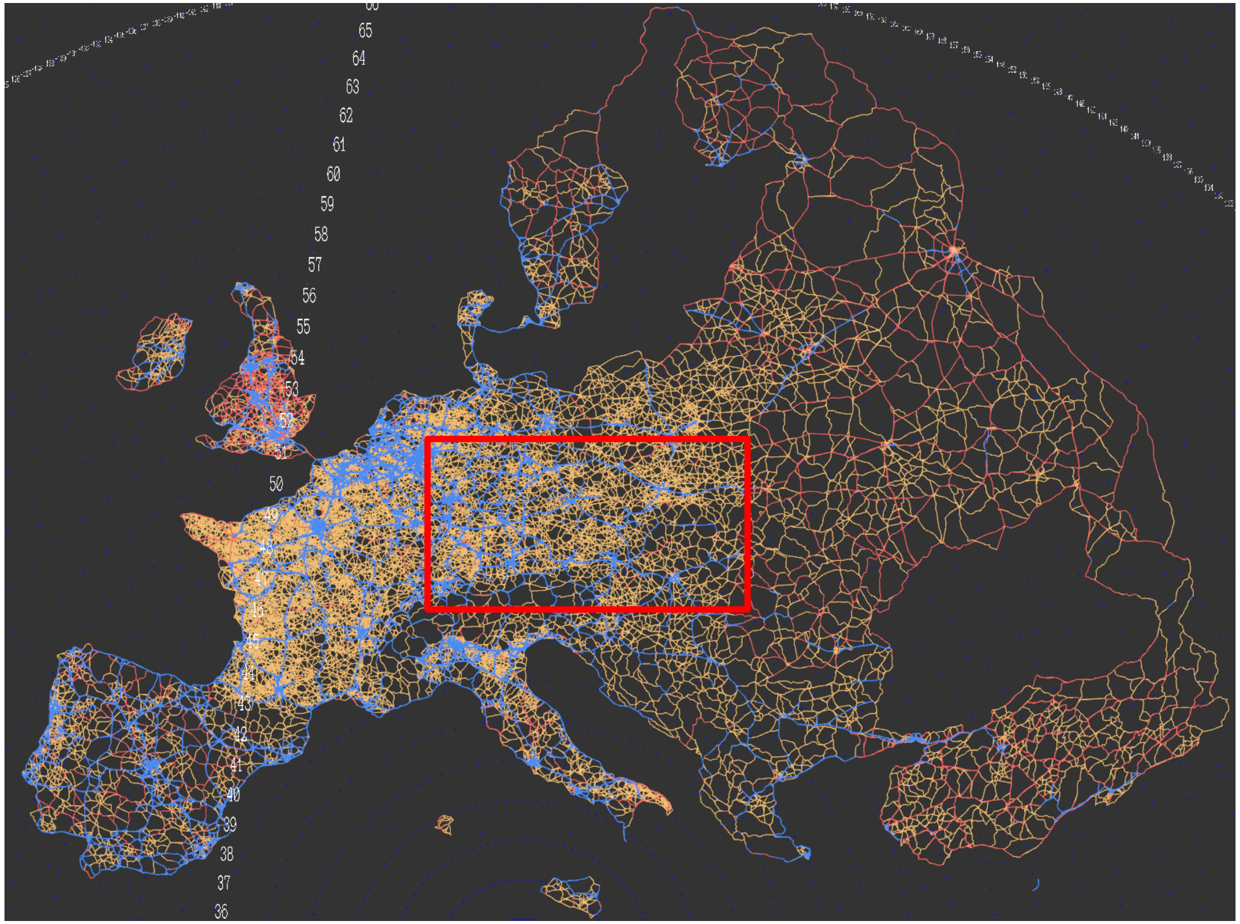
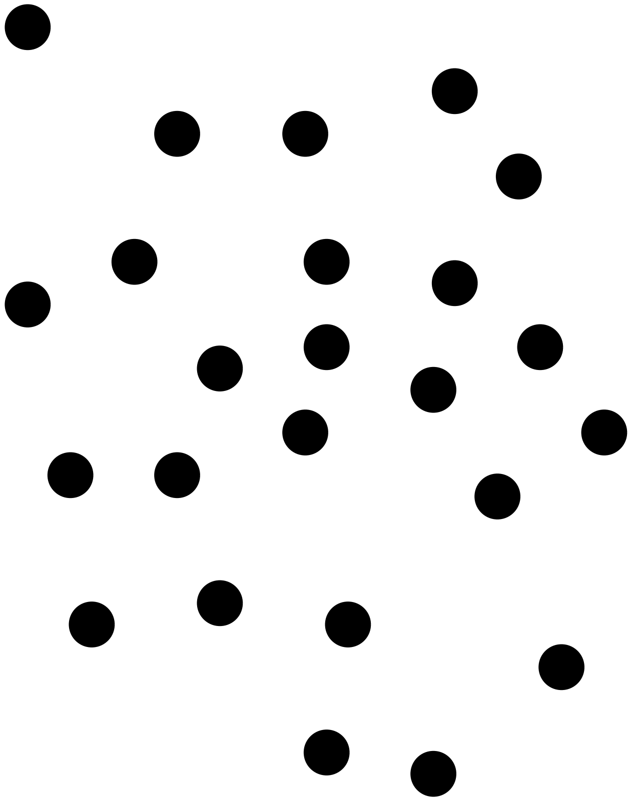
Prioritized Geographic Search

SPP subproject "Lightweight Acquisition and Large-Scale Mining of Trajectory Data"



Prof. Stefan Funke
Institut f. Formale Methoden der Informatik
Abteilung Algorithmik
University of Stuttgart
Germany

Basic Geographic Range Search



Given: Set S of n points in $\mathbb{R}^2$

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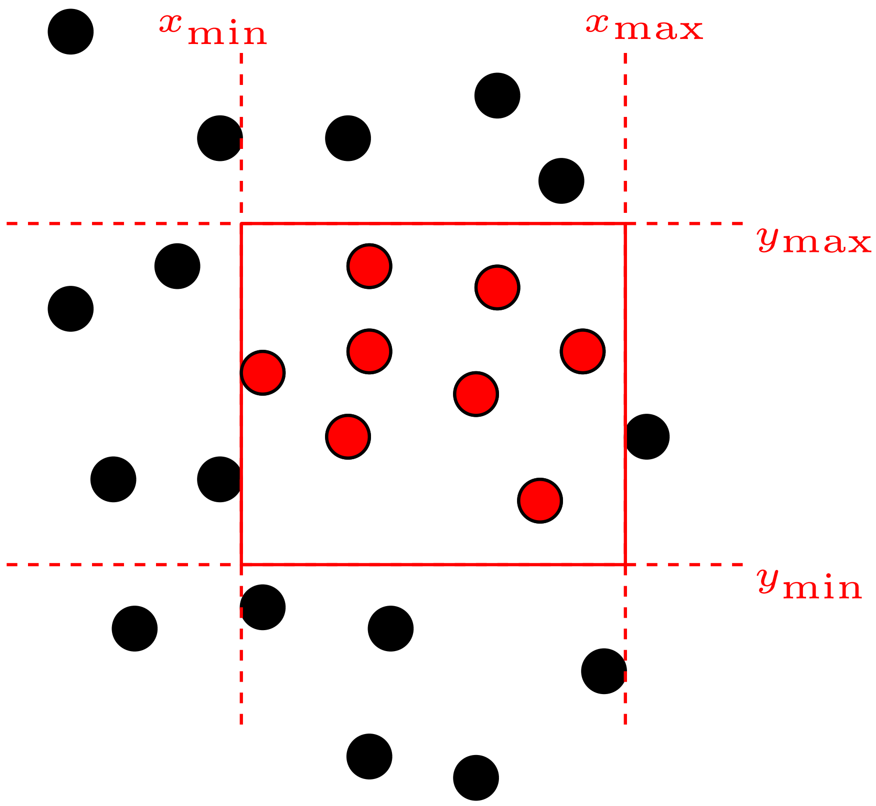

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Goal: Construct data structure $\mathcal{D}$ such that *range queries* specified by $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$ can be answered efficiently.

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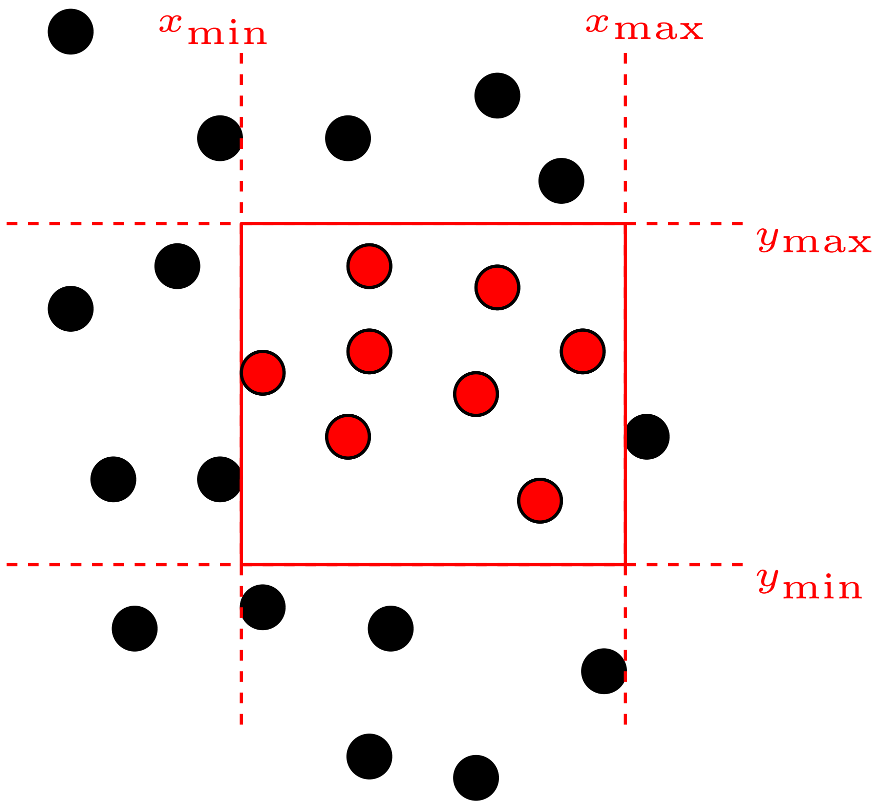

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Examples:
Output within a given area:
- all Greek restaurants
- locations of all mailboxes
- ...

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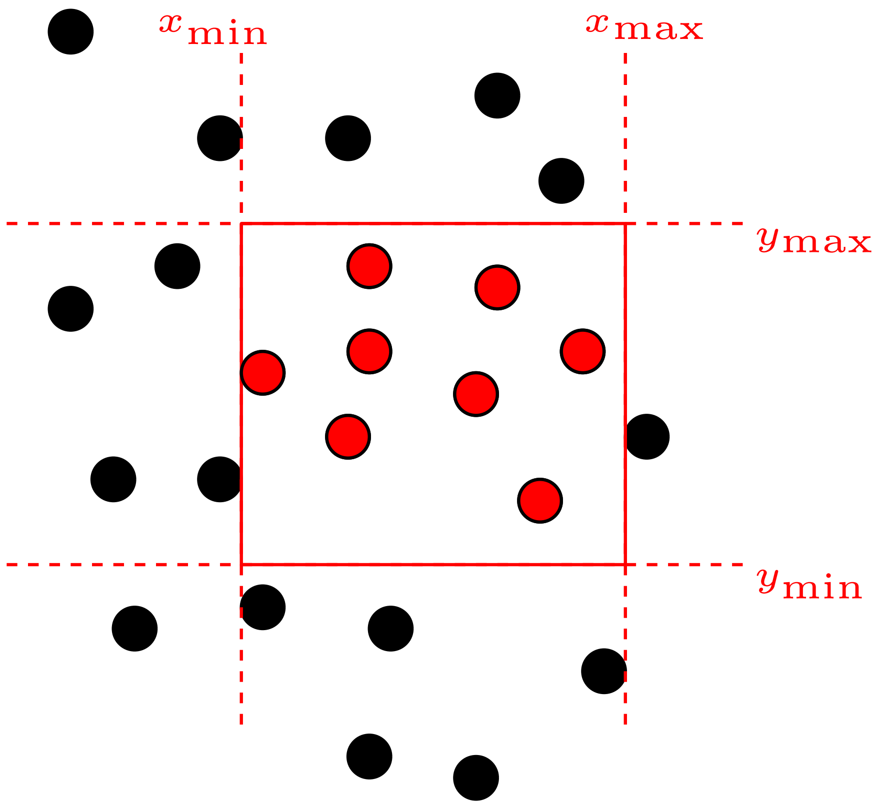

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We care about:
- time $Q(n)$ to answer a single query
- space $S(n)$ of the data structure $\mathcal{D}$
- time $P(n)$ to construct $\mathcal{D}$

Basic Geographic Range Search – Challenges

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Depending on the purpose of the query, it is pretty hard to make sense of this result set . . .

Analogy: Web Search $\approx$ Geographic Search

Web Search:

- Query for "geographic information systems" yields $\approx$ 46 Million results / webpages related to the search term
- yet, we can make sense of the outcome as Google *prioritizes* the outcome in a sensible manner
- good prioritization and prioritized querying main reason for Google prevailing over Yahoo, Altavista, ...

Analogy: Web Search ≈ Geographic Search

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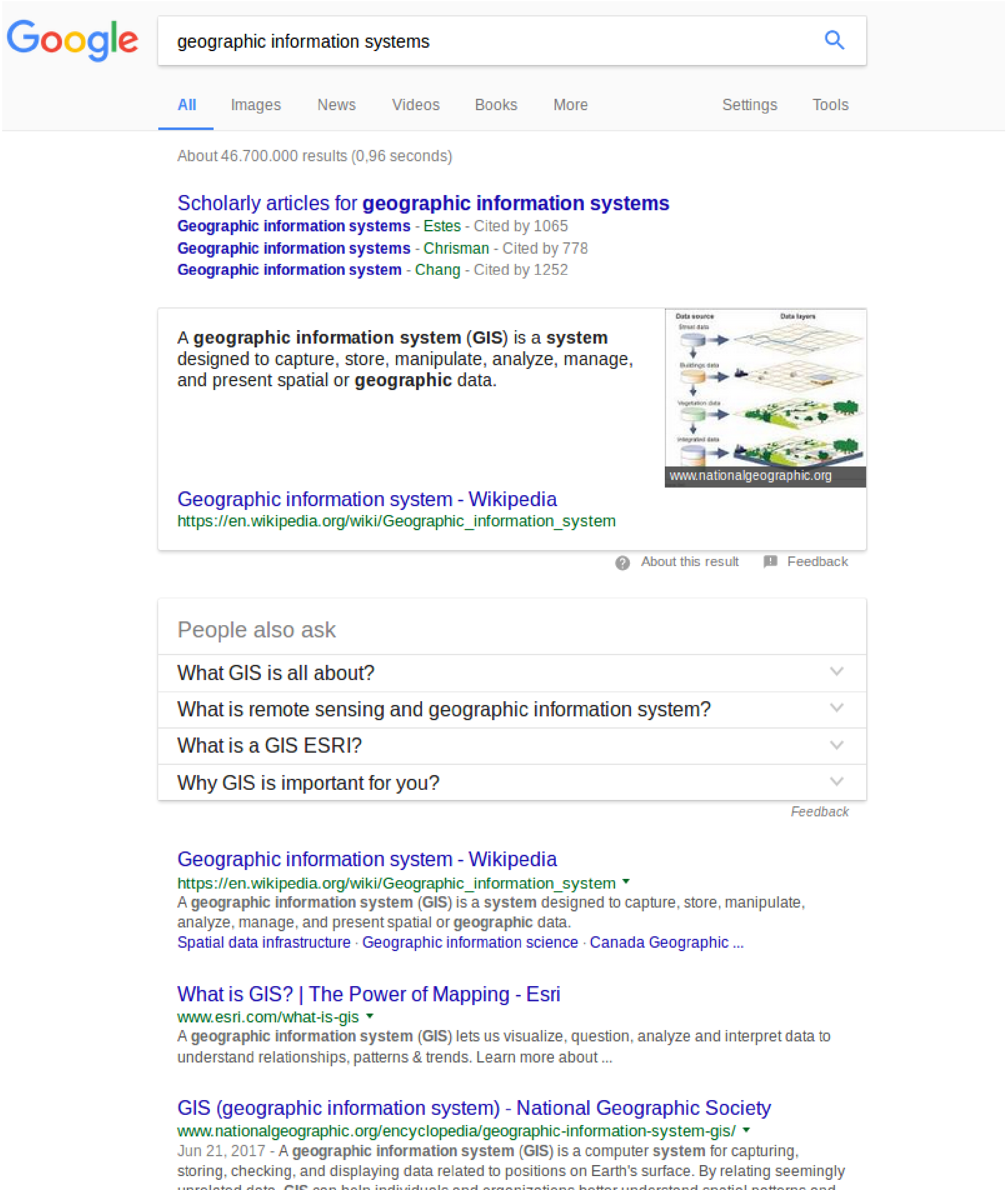

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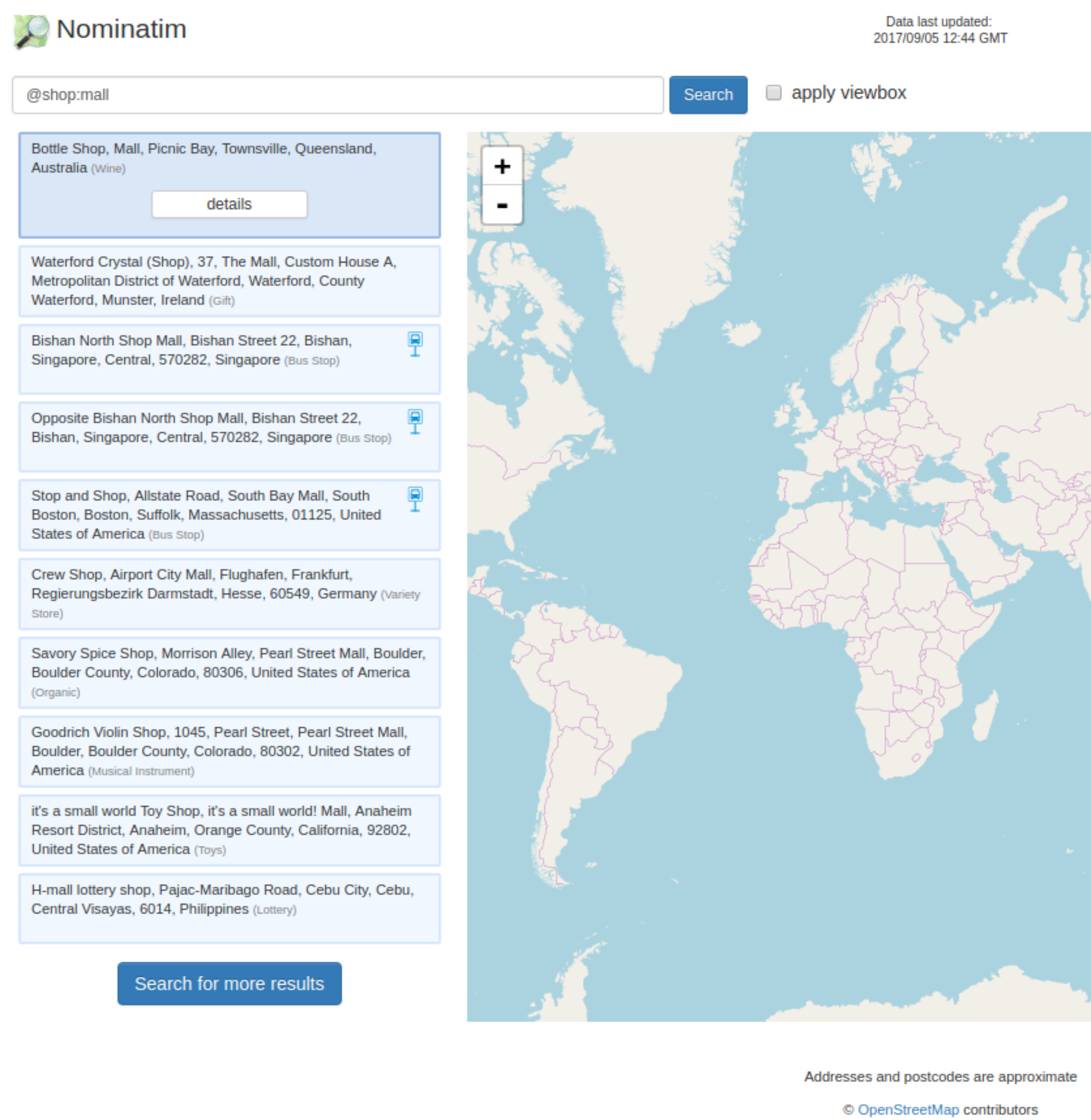
Geographic Search:

- the OpenStreetMap planet data set comprises more than 3 billion items
- an individual query might return hundreds of thousands of items
- again: *prioritization* key for reasonable use and interpretation of results



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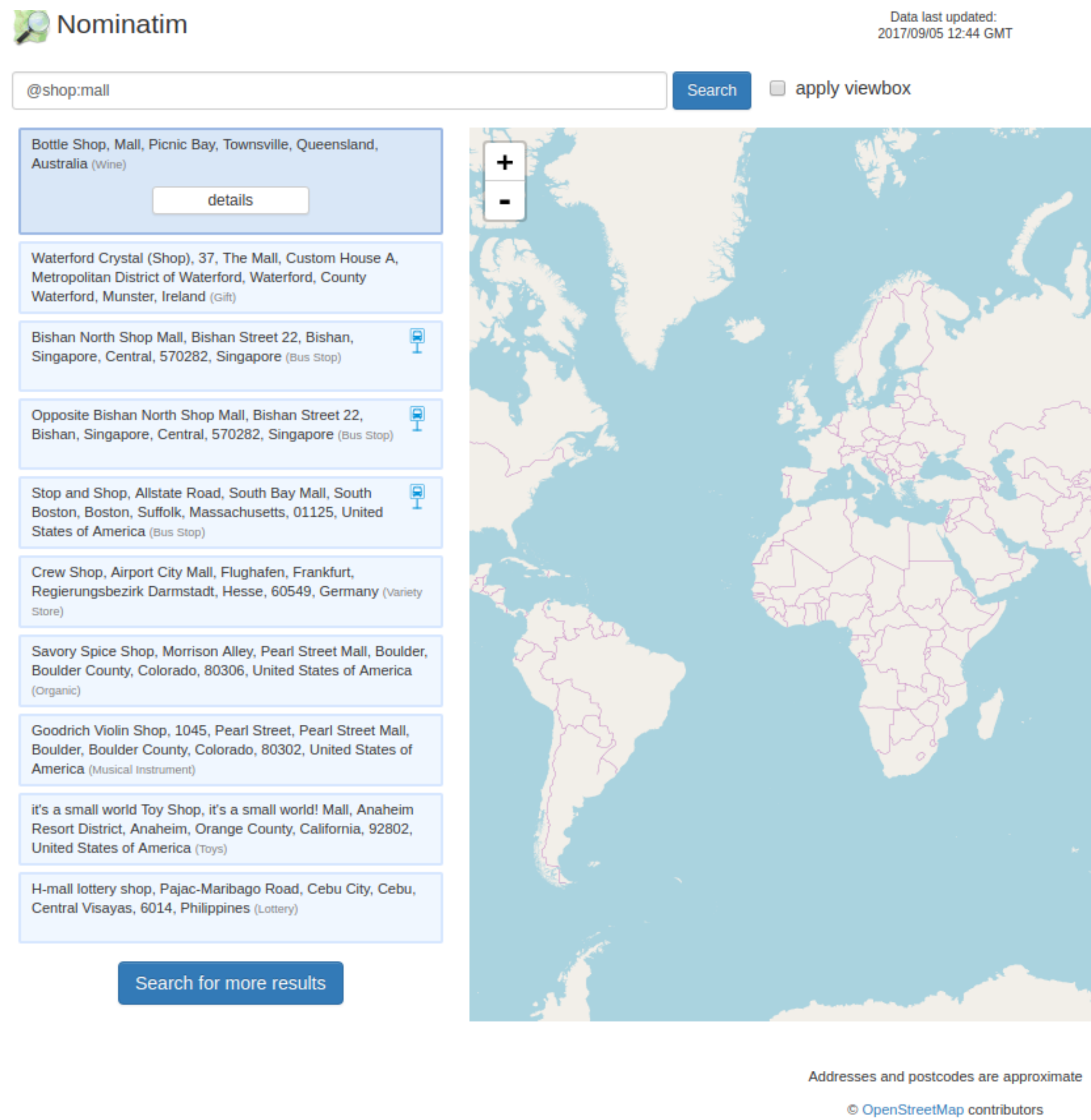
Query: @shop:mall

- ≈ 2,700 in Germany
- prioritization according to size seems very reasonable



Application of Prioritized Range Search: Map Rendering

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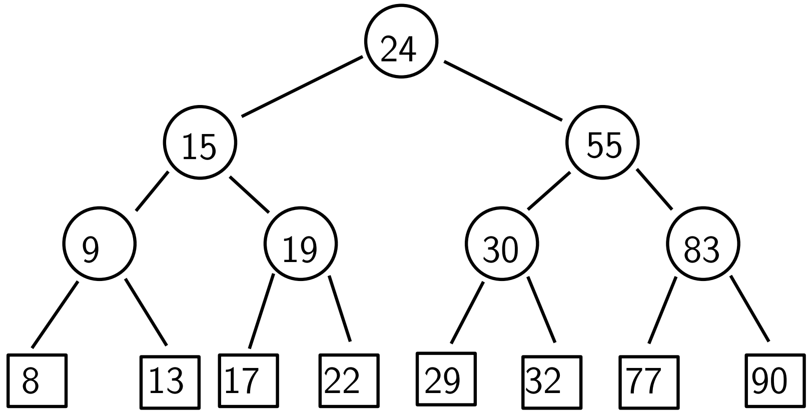
(Almost) direct application of prioritized range queries!

Two-dimensional Range Queries (no priorities yet)

1-dimensional range queries:

- use tree structure
- logarithmic depth
- space consumption for n elements: S(n)=O(n)
- query time for with result size k: Q(n)=O($\log n + k$)



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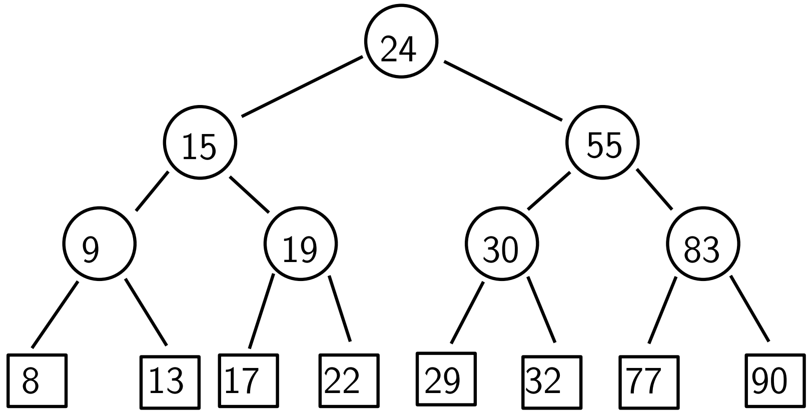

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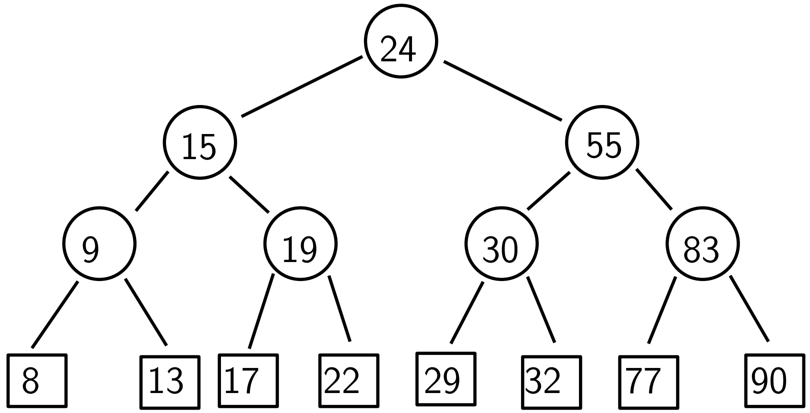

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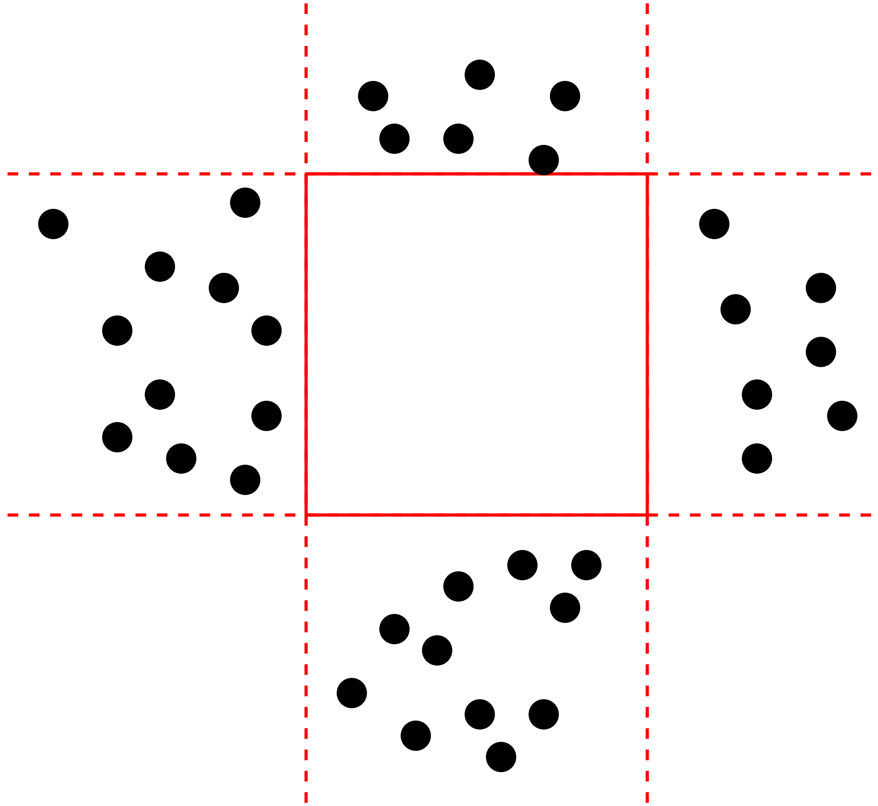
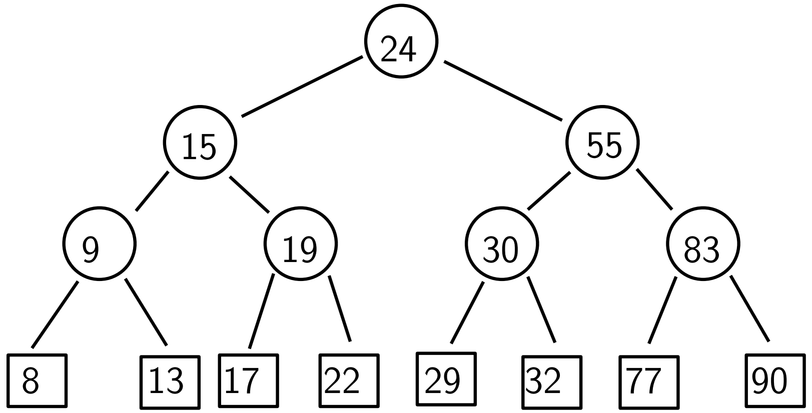


$Q(n)=\Theta(n)$ for a result size of 0!

1st Generalization Attempt: kd-Tree (no priorities yet)

1-dimensional tree construction:

- recursively via median splitting
- construction time $P(n) = O(n \log n)$



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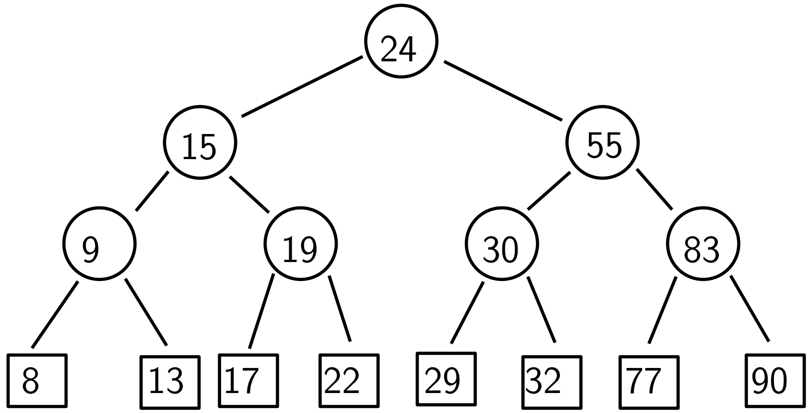

Generalization:

- alternate between median splitting according to x- and y-coordinate
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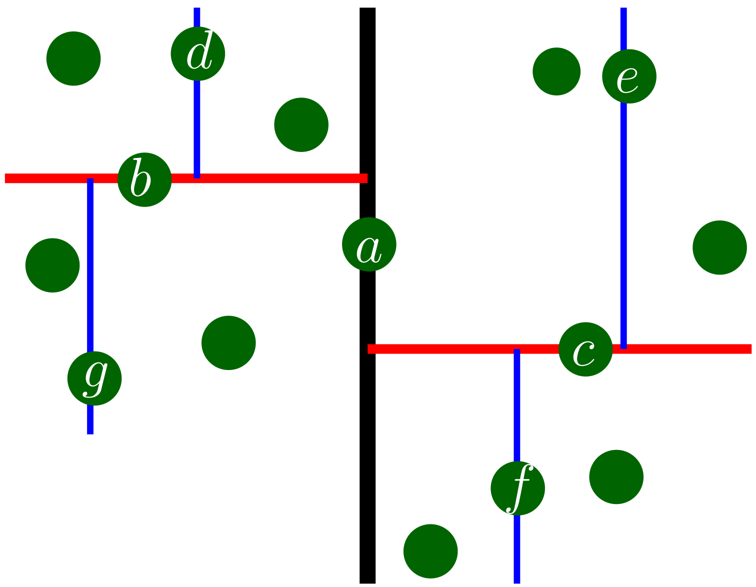

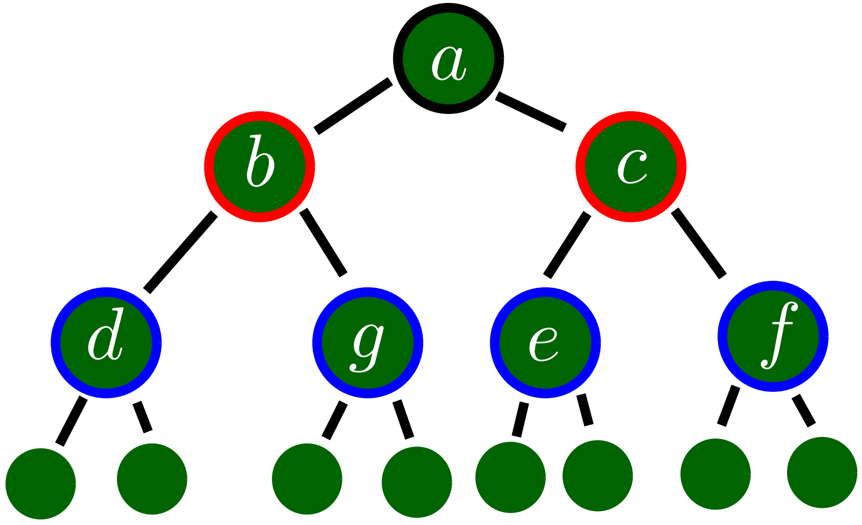

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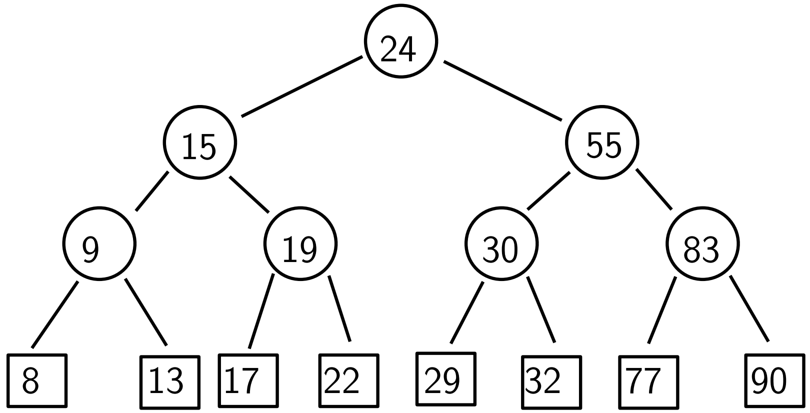

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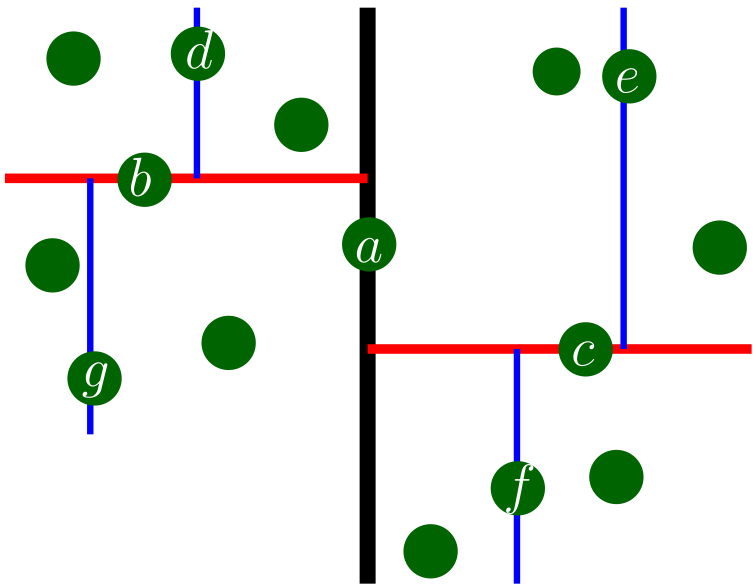

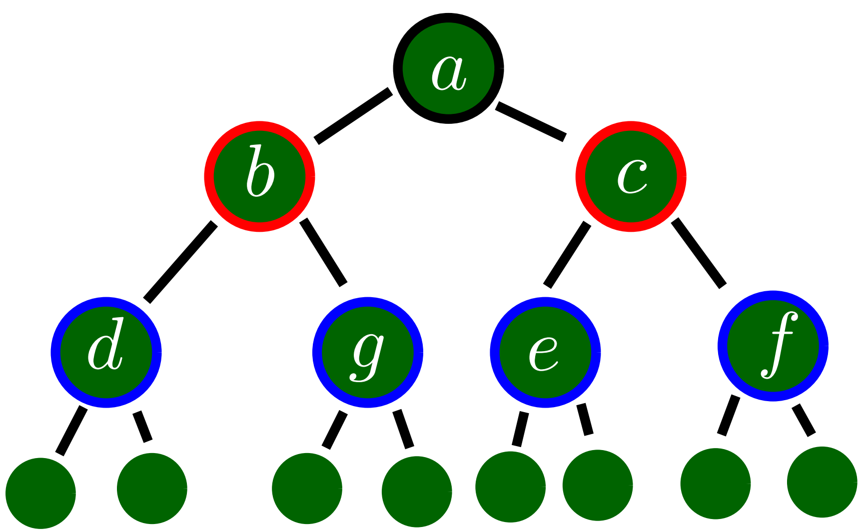

kd-Tree

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Query:

- associate with each node rectangle containing all points of the subtree
- recursively traverse rectangles with non-empty intersection with query rectangle
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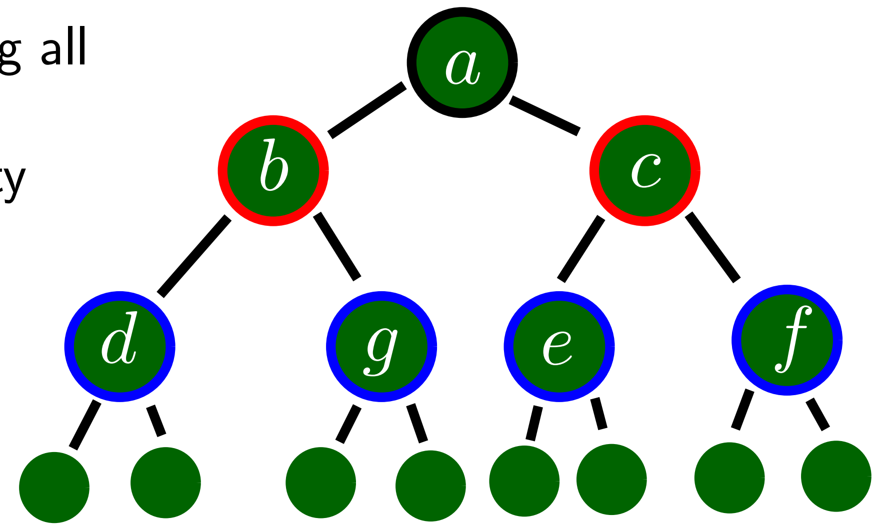

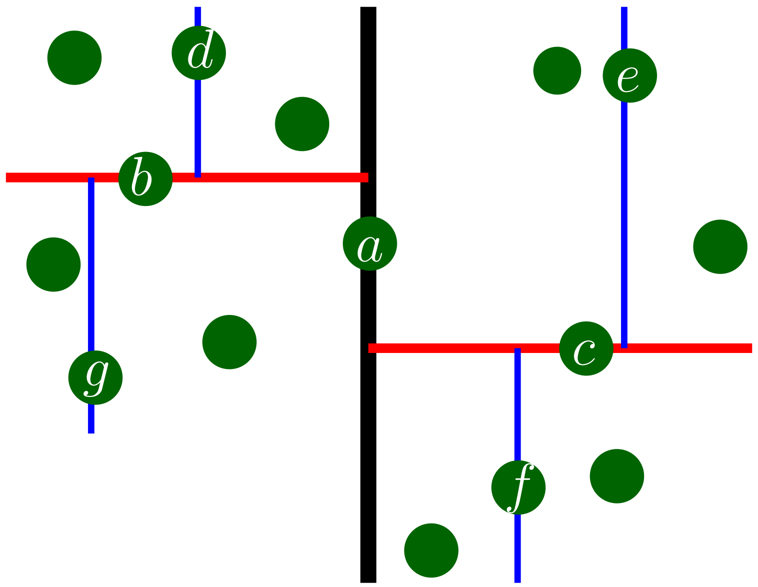

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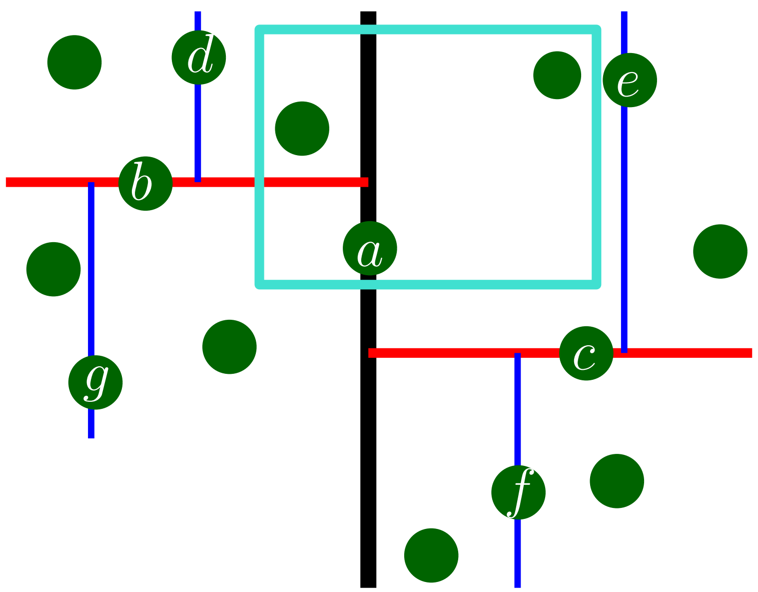

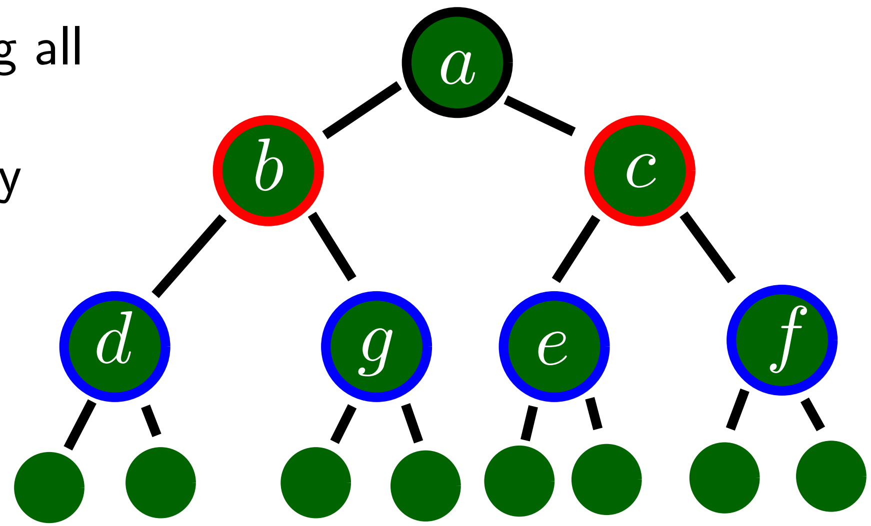

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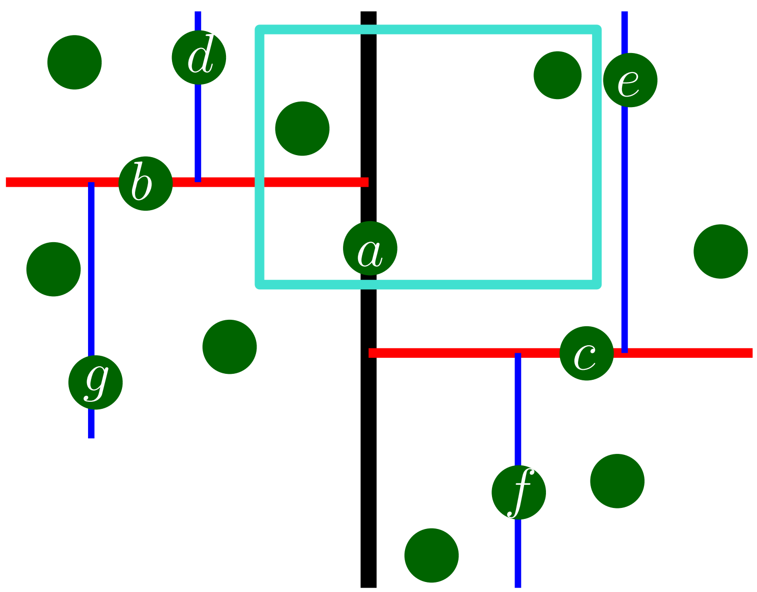

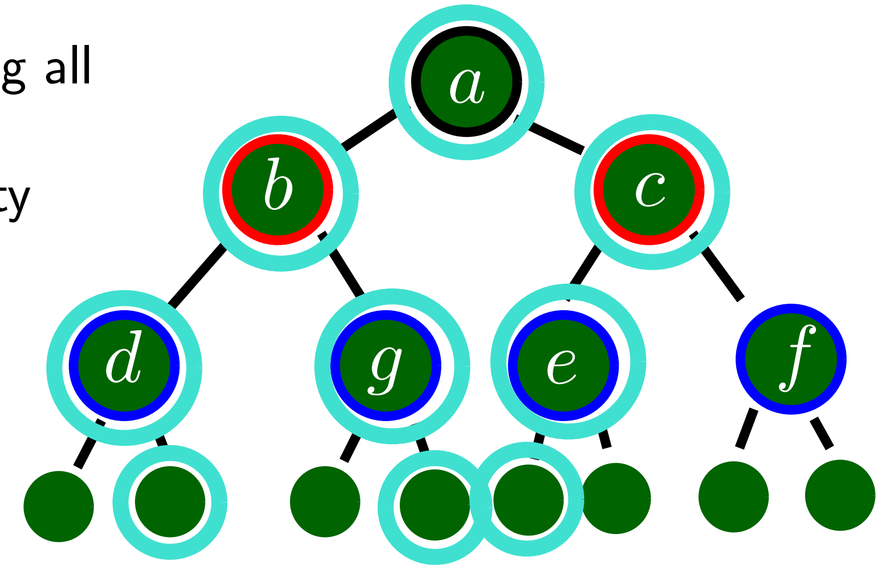

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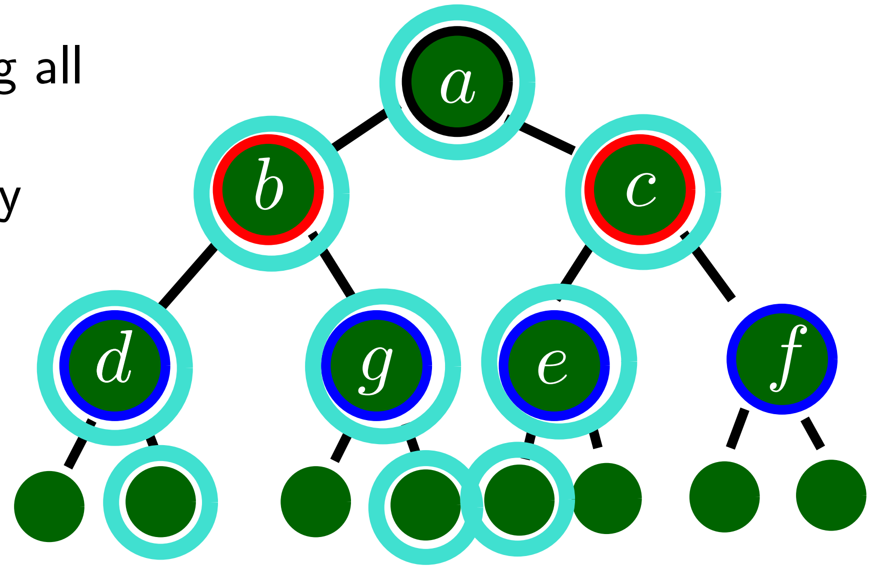
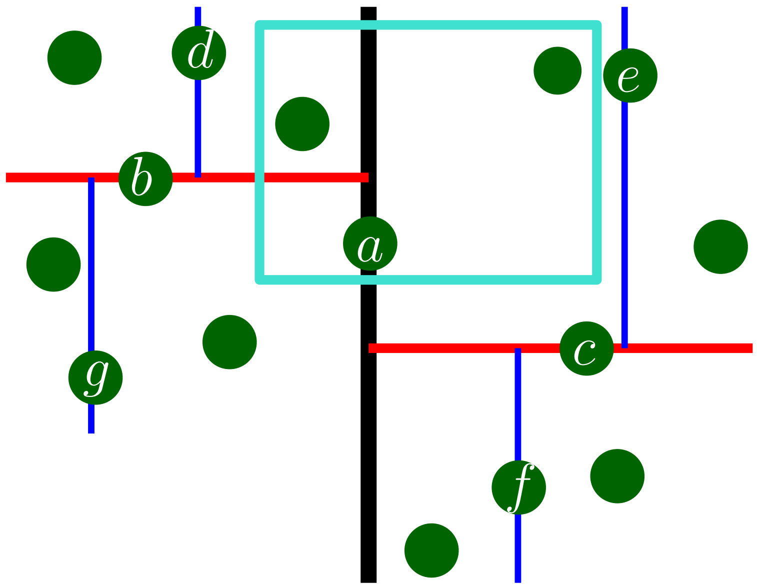
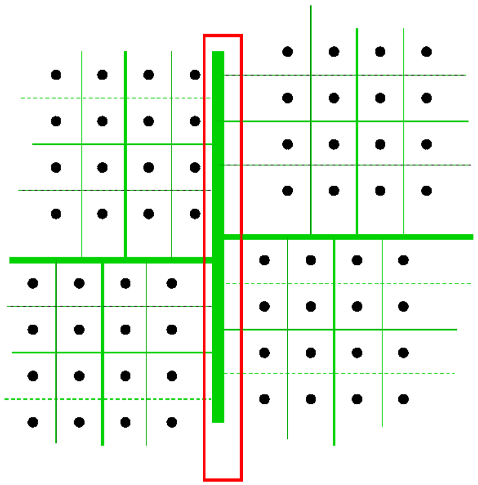




- Query time can be $Q(n) = \Omega(\sqrt{n})$ with 0 points reported :(
- but it cannot be worse $Q(n) = O(\sqrt{n} + k)$



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- every point now also bears a priority
- query is of the form

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We could treat priority as 3rd dimension
(allowing also max-priorities)

- size $S(n) = O(n)$
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Better alternative:

- Augmented kd-prio tree:
  - before median extraction, extract max-priority point and store within node
- space $S(n) = O(n \log n)$
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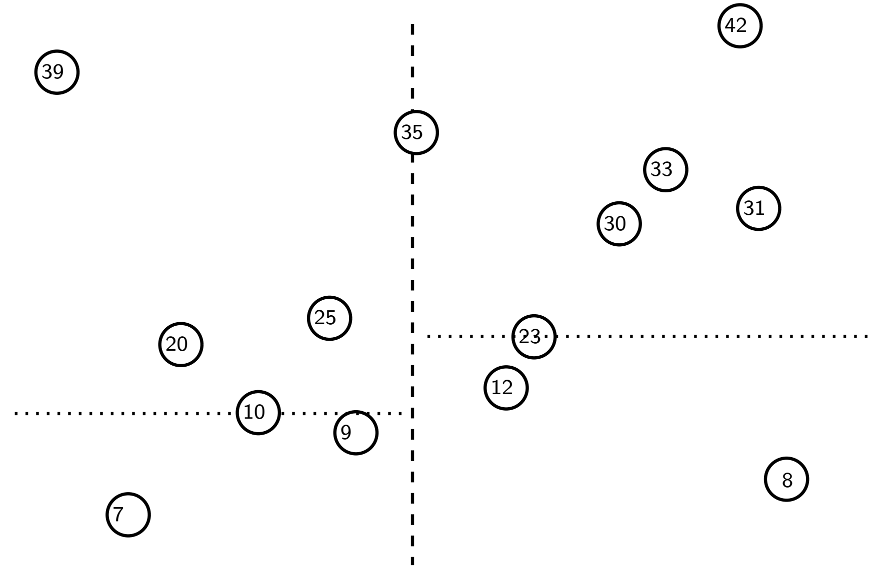

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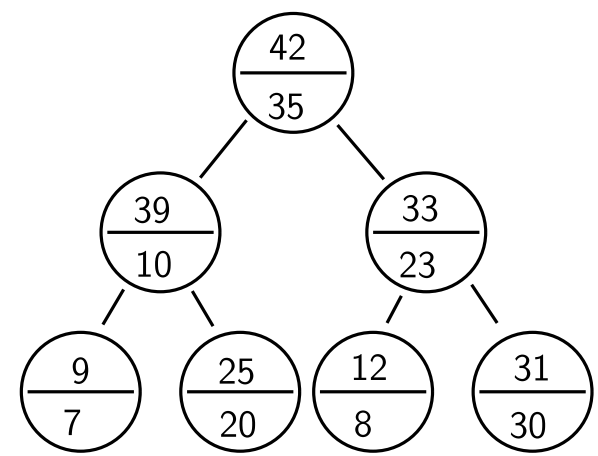

2nd Generalization Attempt: nested RangeTrees

- organize points in 1-dimensional search tree according x-coordinate
- for query $[x_{\min}, x_{\max}] \times [y_{\min}, y_{\max}]$:
  - search for $x_{\min}$ and $x_{\max}$ in tree
  - observation: all points with matching x-coordinate 'enclosed' by search paths
- **store in each internal node all points of subtree in a tree on the y-coordinates**
- space $S(n) = O(n \log n)$
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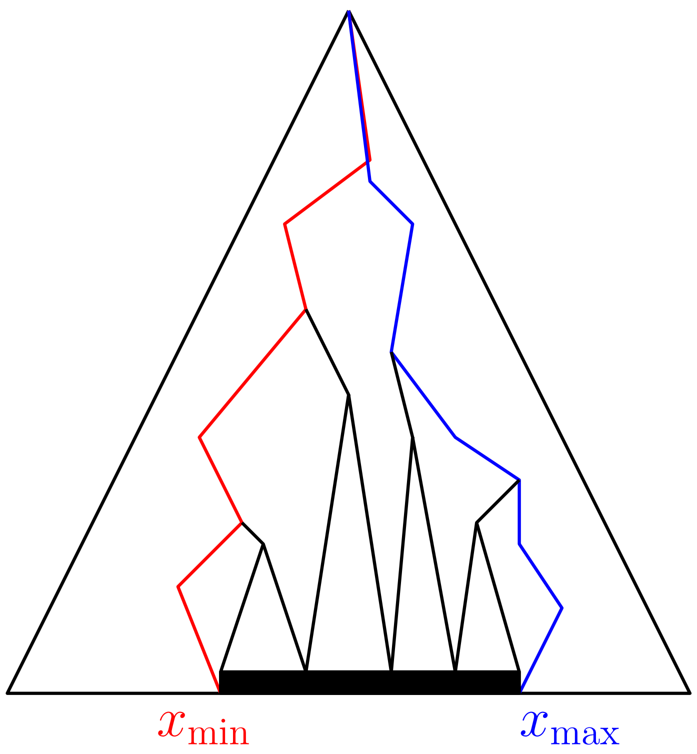

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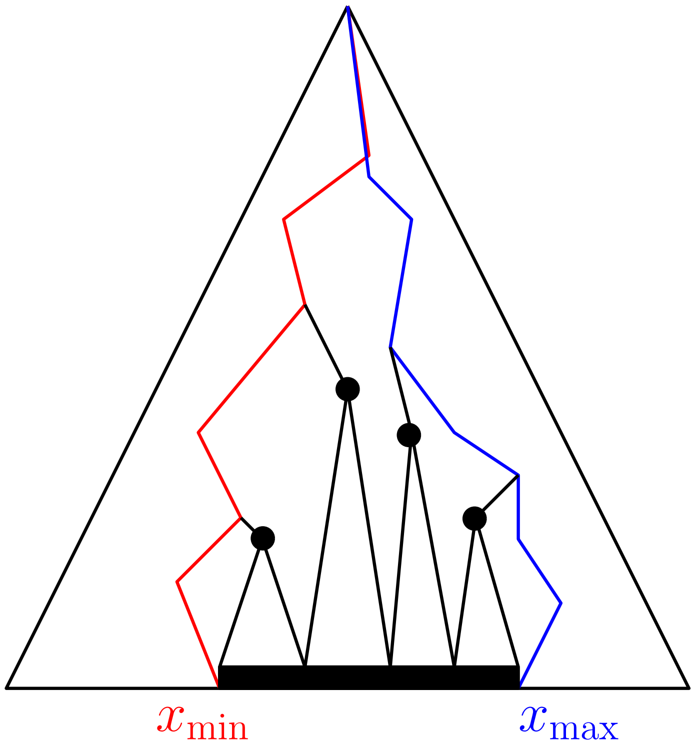

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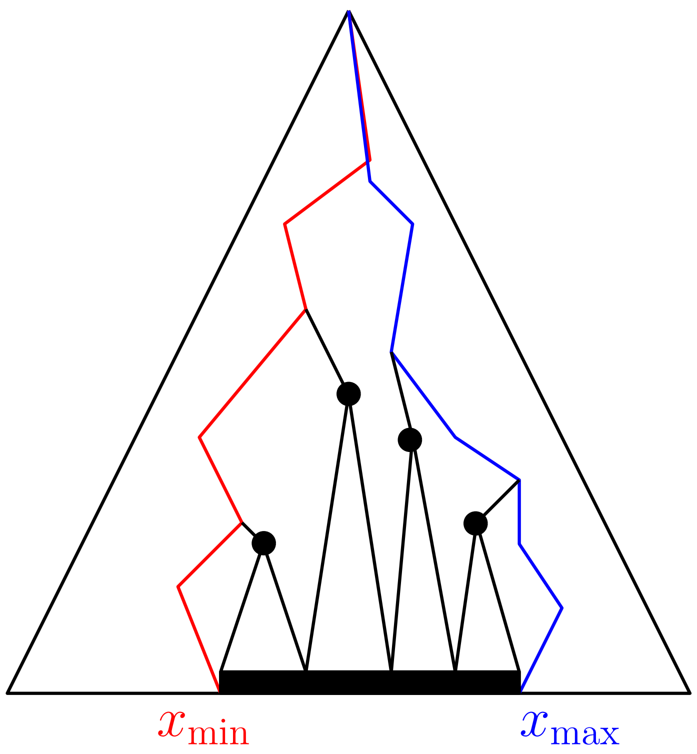

Priorities can be incorporated as 3rd dimension:

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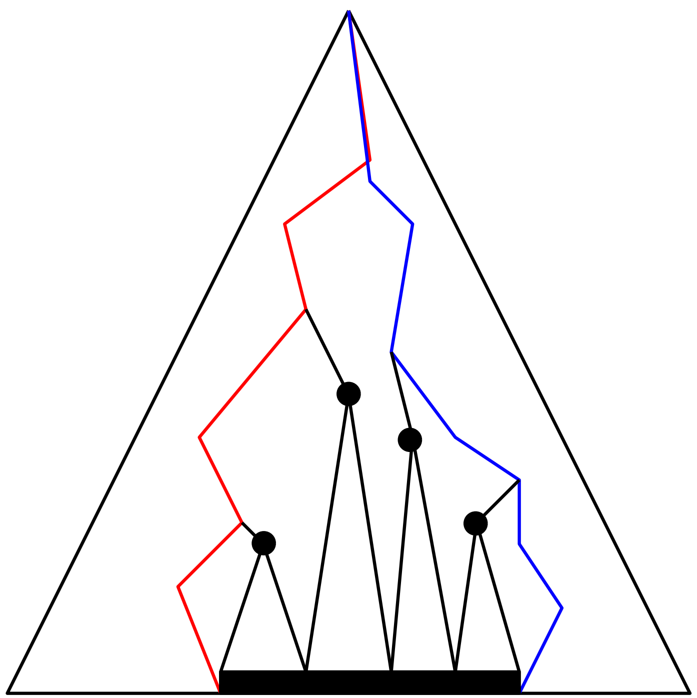

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We can do better by using another secondary structure . . .

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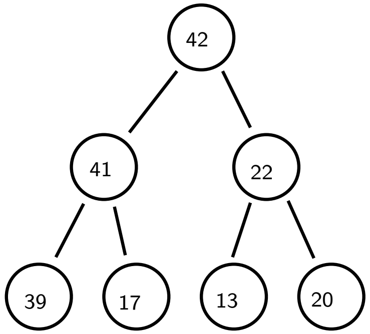
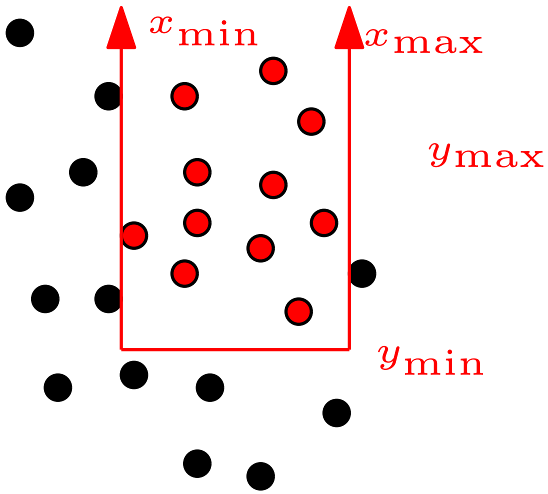
2nd improved Generalization Attempt: Range Tree+Treap

- A *treap* can answer queries of the type

$$[x_{\min}, x_{\max}] \times [y_{\min}, \infty]$$

- *treap*$\equiv$ *tree* and *heap* at the same time





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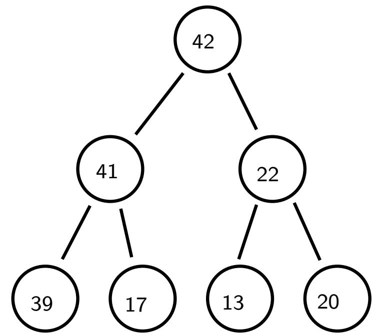

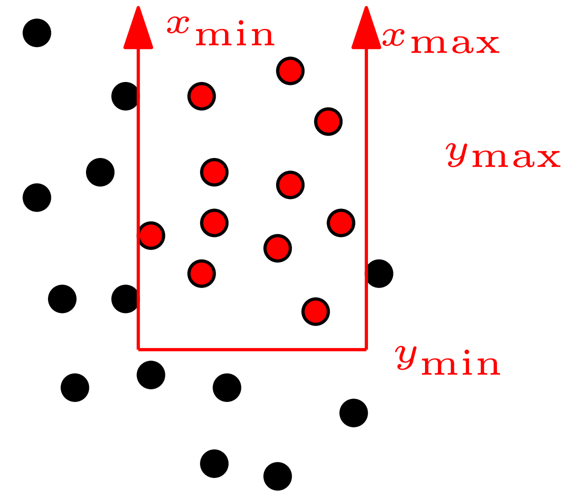

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- extract max-prio point and x-median point
- recursively construct left and right subtrees
- time $P(n) = O(n \log n)$
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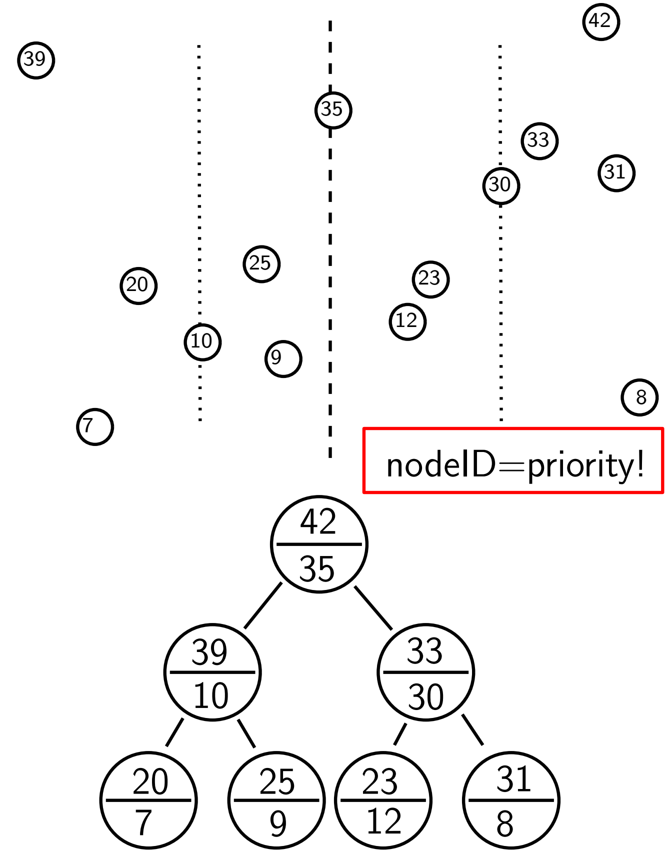

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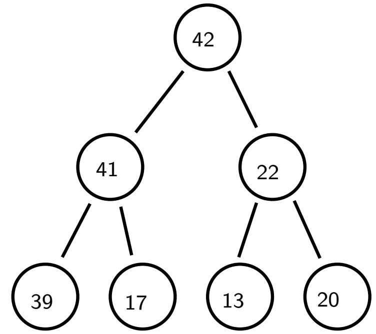

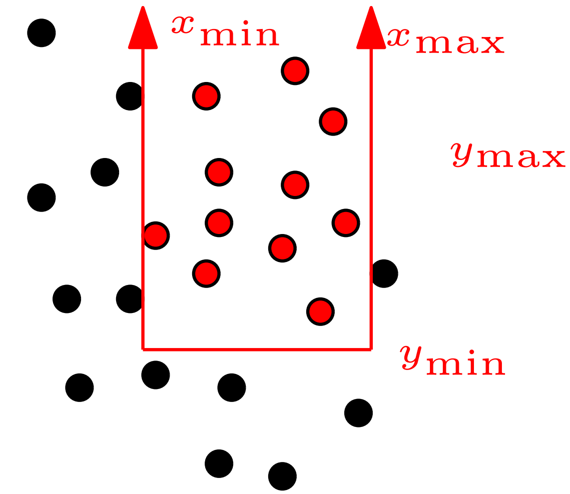

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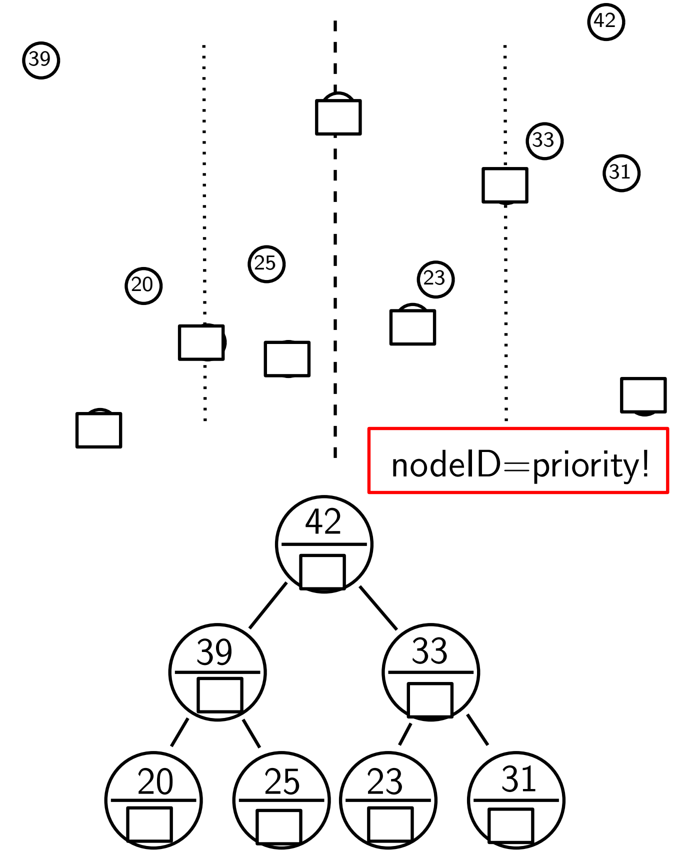

2nd improved Generalization Attempt: Range Tree+Treap

- A *treap* can answer queries of the type

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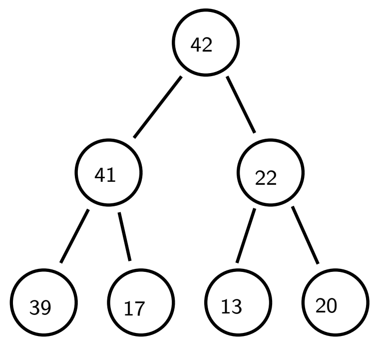
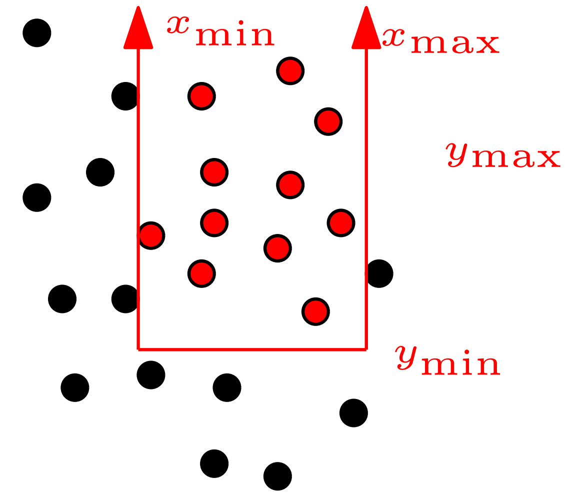
- *treap*$\equiv$ *tree* and *heap* at the same time





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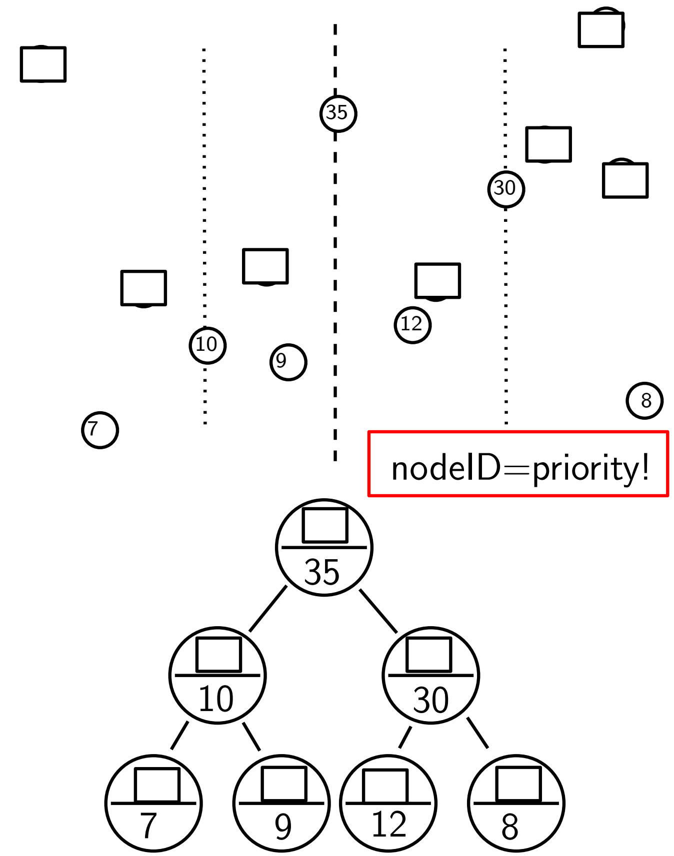

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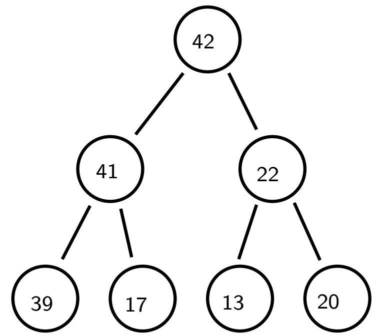

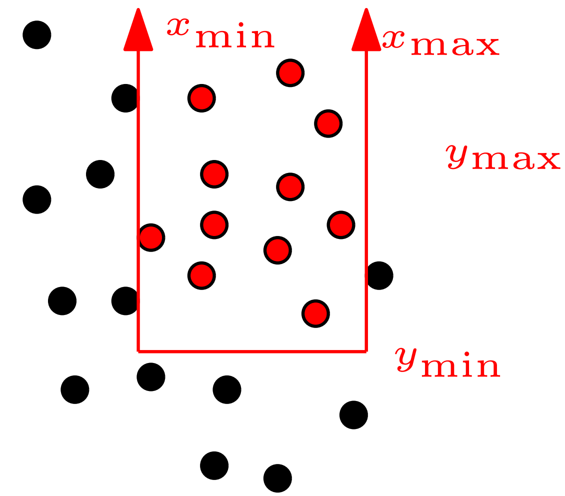

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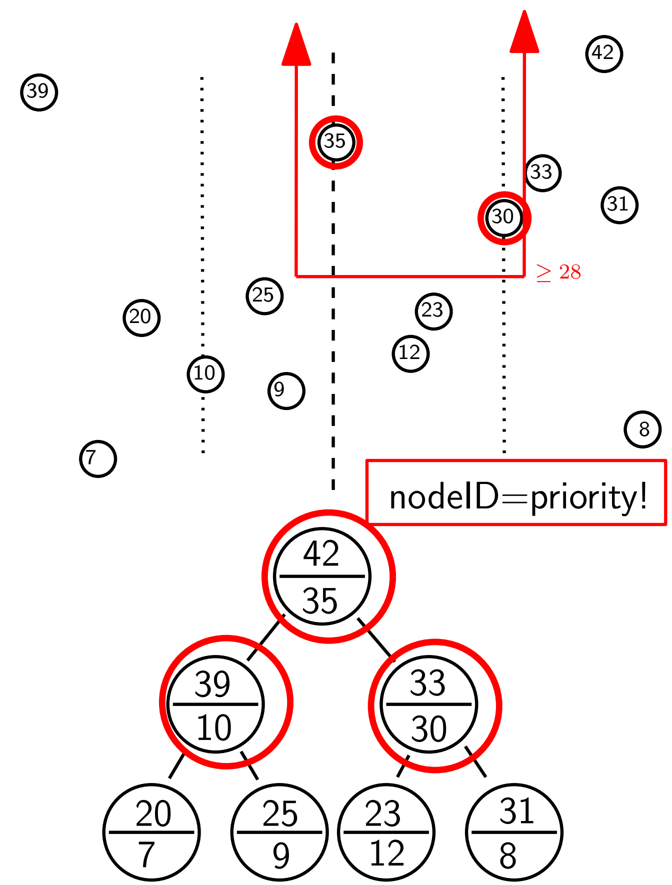

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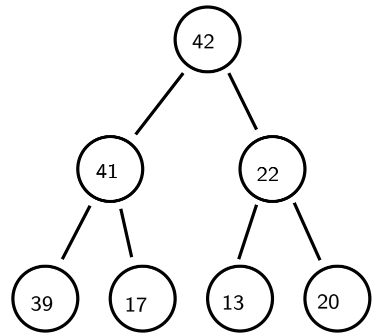

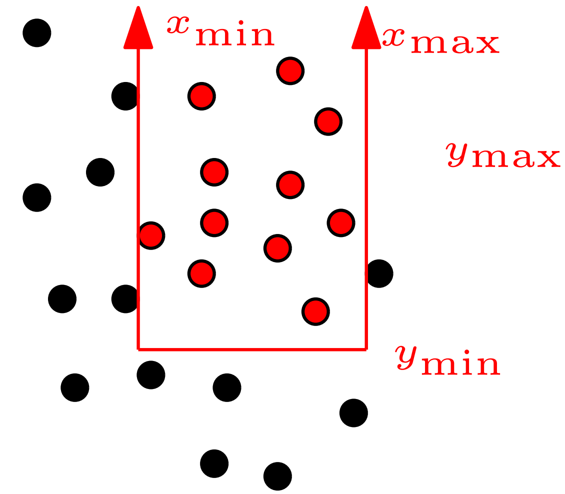

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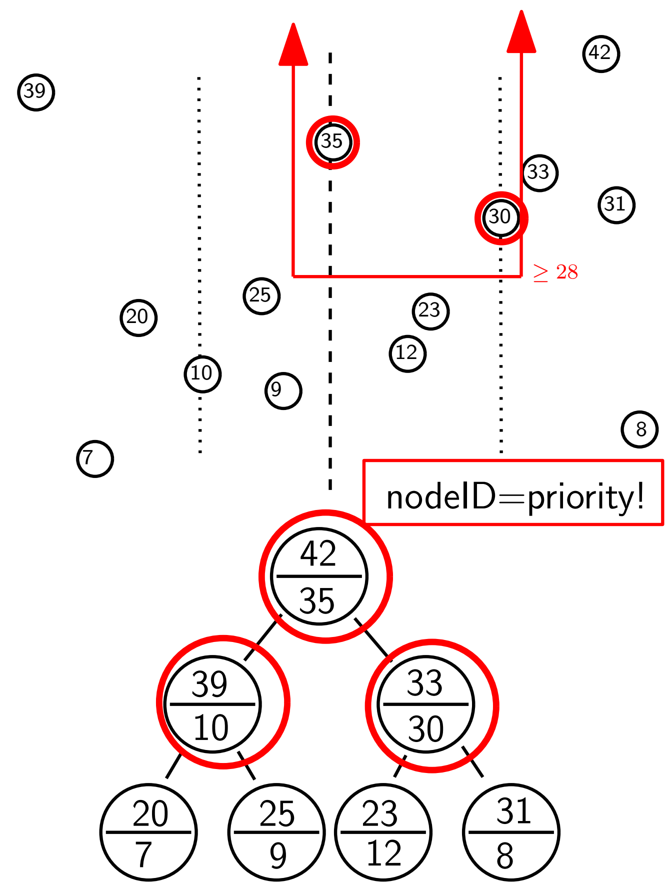

Use treap as secondary structure in a 1-dimensional range tree:

- $S(n) = O(n \log n)$
- $Q(n) = O(log^2 n + k)$

Things to consider for 'Practical' Solutions on large Data Sets

- use only space *linear* in the number of data items

Example:

- $n = 2^{29}$ (around 500 million)
- $n \log n = 29 \cdot n$, that is, we need 29 times more space than just to store the data!

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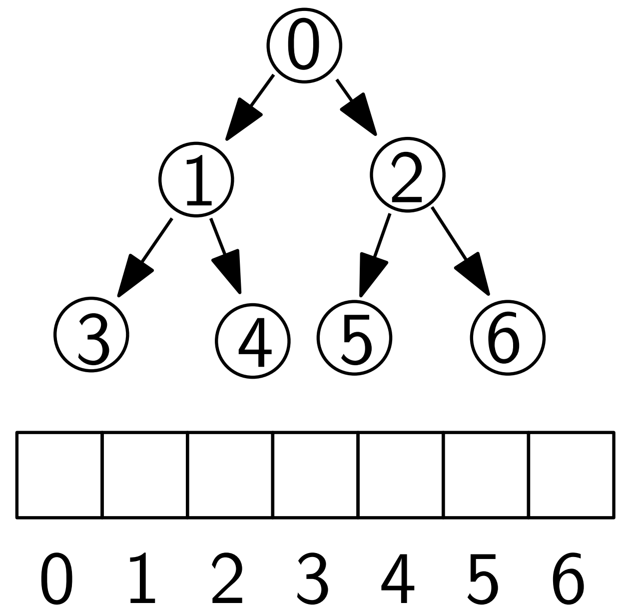

Example:

- data item has 10 bytes (lat/lon/prio)
- every node in the tree must have incoming pointer (8 Bytes)
- connectivity information almost *doubles* space
- Better: implicit connectivity e.g. via array representation:
  - leftchild(i)$=2 \cdot i + 1$
  - rightchild(i)$=2 \cdot i + 2$
  - parent(i)$=i/2$

Challenge:

- need 'almost complete' search tree
- requires more deliberate splitting choice than median

Things to consider for 'Practical' Solutions on large Data Sets

- use only space *linear* in the number of data items
- try to avoid explicit storage of connectivity information
- try to keep things in memory consecutively

More concrete:

- storing and scanning elements linearly in an array is surprisingly fast (cache effects)
- tree search issues rather dispersed memory accesses
- avoid many individual 'objects' allocated on the heap
- often a combination of a 'real' data structure with linear search very effective

A simple, yet practical solution: Grid + Linear Scan

Linear Scan:

- store (lat,lon,prio) consecutively in an array
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- create grid
- store grid cells consecutively in memory
- within each grid cell store items sorted according to prio
- for query:
  - determine grid cells to be inspected
  - perform prioritized linear scan in each of them
- grid: limits effort if query region small
- prio sort: limits effort if min priority high
- parameter to choose: cell size
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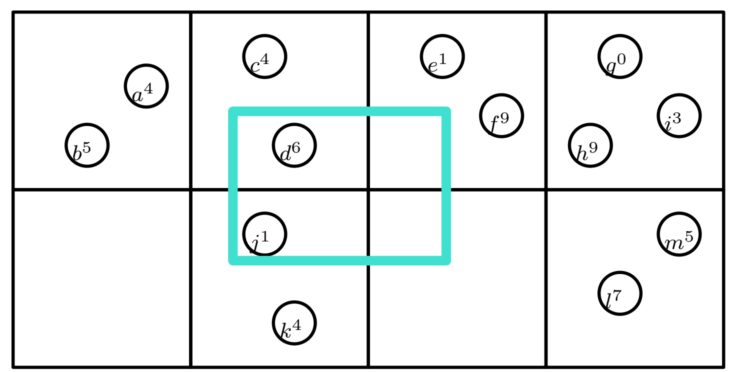

$$b^5 \quad a^4 \quad d^6 \quad c^4 \quad f^9 \quad e^1 \quad h^9 \quad i^3 \quad g^0 \quad k^4 \quad j^1 \quad l^7 \quad m^5$$

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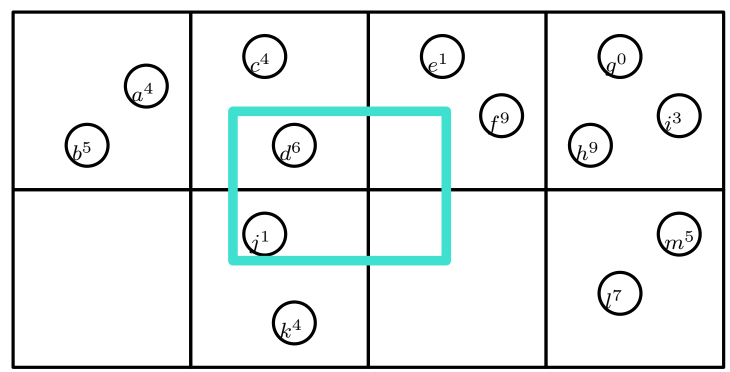

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You will implement this in the exercises!

Thank you

... for your attention!

Questions?

Applications? ($\rightarrow$ also talk to Filip!)

See you in the exercise session . . .

In Detail: Bounds for kd-Tree Query Time

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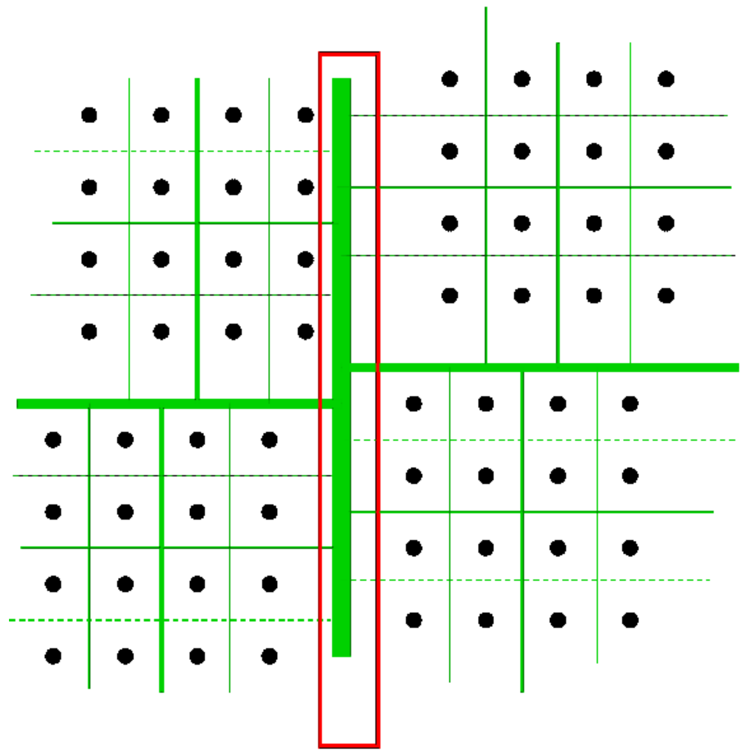
. . . in fact pretty bad:



In this perturbed 8×8 grid we have $n = 64$ and the query box intersects $\Omega(\sqrt{n})$ many cells without reporting anything.

$$\Rightarrow S(n) = \Omega(\sqrt{n} + k)$$

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Fortunately, NO!
We can prove a bound of

$$Q(n) = O(\sqrt{n} + k)$$

due to the following observations:

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How many cells can be intersected by one vertical line?

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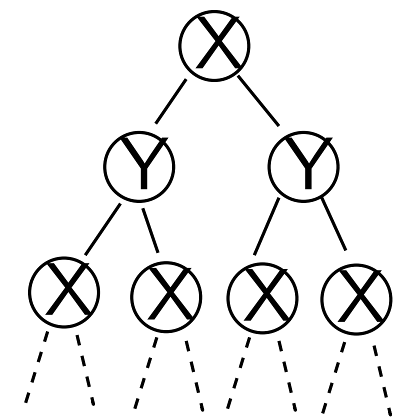

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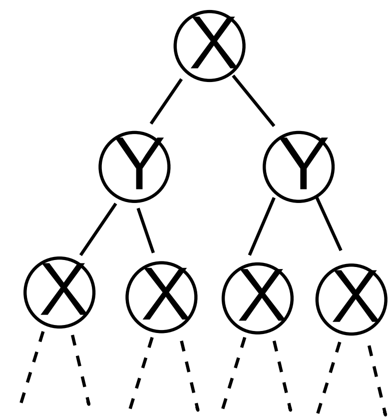

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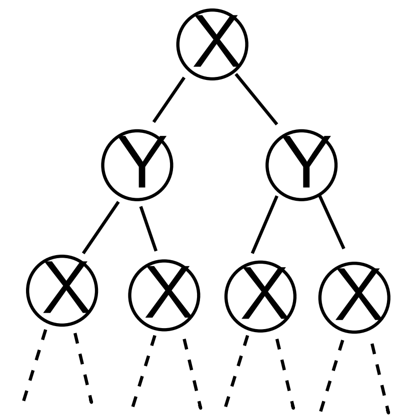

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